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B. E. (Third Semester) Examination, Nov.-Dec. 2021

(New Scheme)

(IT Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note *: Attempt all questions. Part (a) is compulsory from each unit & carrying 2 marks. Solve any other two parts from (b), (c) and (d) questions from each unit & carrying 7 marks.*

Unit-I

1. (a) State dirichelet's condition for fourier expansion.

(b) Find the fourier series of the periodic function $f(x)$ where

$$f(x)=\begin{cases}-\pi & \text{when} \quad -\pi<x<0 \\ x & \text{when} \quad 0<x<\pi\end{cases}$$

Hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots$

(c) Find the fourier series for the function $f(x)$, of $f(x)=|\cos x|$ on $-\pi<x<\pi$.

(d) The following table gives the variations of periodic current over a period

t sec	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A amp	1·98	1·30	1·05	1·30	– 0·88	– 0·25	1·98

Show that there is a direct current part of 0·75 amp in the variable current and obtain the amplitude of the first harmonic.

Unit-II

2. (a) Write the laplace transform of periodic function of $f(t)$ with period T.

(b) (i) Evaluate $\int_0^{\infty} \frac{e^{-at}-e_{t}^{-bt}}{t}\,dt$ 4

(ii) Show that $\int_0^{\infty} te^{-2t}\cos t\; dt=\frac{3}{25}$ 3

(c) Prove that $L^{-1}\left\{\frac{S}{S^4+S^2+1}\right\}=\frac{2}{\sqrt{3}}\sinh\frac{t}{2}\sin\frac{\sqrt{3}t}{2}$.

(d) Solve $(D^2+9)x=\cos 2t$ if $x(a)=1$, $x\left(\frac{\pi}{2}\right)=-1$.

Unit-III

3. (a) Write Cauchy Integral Formula.

(b) If $w=f(z)=u+iv$ is analytic function and $u-v=e^x$ (cosy – siny). Find w in terms of z.

(c) Evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)}\,dt$ where C is the order $|z|=3$.

(d) Show that $\int_0^{\pi} \frac{\cos 2\theta d\theta}{1-2a\cos\theta + a^2} = \frac{\pi a^2}{1-a^2} \quad (a^2 < 1)$.

Unit-IV

4. (a) Eliminate the arbitrary function from the relation

$z = e^{ny} f(x-y)$.

(b) Solve :

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$$

(c) Solve :

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial x \partial y} - 6\frac{\partial^2 t}{\partial y^2} = y\cos x$$

(d) Using the method of separation of variables solve.

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

given that $u(x,0) = 6e^{-3x}$.

Unit-V

5. (a) Define moment generating function.

(b) The probability density $p(x)$ of a continuous random variable is given by :

$$p(x) = y_0 \, e^{-|x|} dx \quad -\infty < x < \infty$$

prove that $y_0 = \frac{1}{2}, \mu_1' = 0, \ \sigma = \sqrt{2}$ and mean deviation about mean is 1.

(c) The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that :

(i) Exactily two will be defective

(ii) At least two will be defective

(iii) None will be defective

(d) Fit poisson's distribution to the following and calculate theoretical frequency $(e^{-0.5} = 0.61)$.

Deaths	:	0	1	2	3	4
Frequency	:	122	60	15	2	1